

# Chapter 4 Section 4

## Radical Expressions

### Square roots of perfect squares

- A perfect square is an integer multiplied by itself.

### Examples of perfect squares:

- 1, 4, 9, 16, 25, and 36 are examples of perfect squares because

$$\begin{array}{ll} 1^2 = 1 & 4^2 = 16 \\ 2^2 = 4 & 5^2 = 25 \\ 3^2 = 9 & 6^2 = 36 \end{array}$$

Note that squaring the negative integers results in the same list of numbers

$$\begin{array}{ll} (-1)^2 = 1 & (-4)^2 = 16 \\ (-2)^2 = 4 & (-5)^2 = 25 \\ (-3)^2 = 9 & (-6)^2 = 36 \end{array}$$

The square root of a number (X) is a number (a) which when multiplied by itself ( $a^2$ ) gives us X

$$\sqrt{X} = a \quad \text{if} \quad a^2 = X$$

### Examples of square roots:

Simplify  $\sqrt{49}$

$$7^2 = 49 \quad \text{so} \quad \sqrt{49} = 7$$

Simplify  $\sqrt{25} + \sqrt{81}$

$$5^2 = 25 \quad \text{and} \quad 9^2 = 81$$

$$\begin{aligned} \text{so } \sqrt{25} + \sqrt{81} &= 5 + 9 \\ &= 14 \end{aligned}$$

## Square roots of whole numbers:

When taking the square root of a number that is not a perfect square, the answer can only be estimated.

### Examples:

Between what two whole numbers is the value of  $\sqrt{41}$  ?

Since  $6^2 = 36$  and  $7^2 = 49$  then the  $\sqrt{41}$  must be between 6 and 7.

Between what two whole numbers is the value of  $\sqrt{14}$  ?

Since  $3^2 = 9$  and  $4^2 = 16$ , then the  $\sqrt{14}$  must be between 3 and 4.

### Approximating the square root of a number to the nearest tenth:

Step 1 – get the difference between the two perfect squares closest to the number, one greater than and one less than the number.

Step 2 – get the difference between the number we are trying to take the square root of and the perfect square that is less than the number.

Step 3 – now divide the difference between the two perfect squares (found in step 1) by the difference between the perfect square and the number (found in step 2).

Step 4 – add the answer from step 3 to the square root of the smaller perfect square.

**Example:** Approximate  $\sqrt{59}$  to the nearest tenth.

The  $\sqrt{59}$  is between the  $\sqrt{49}$  (the closest perfect square less than 59) and  $\sqrt{64}$  (the closest perfect square greater than 59)

Step 1 – the difference between the perfect squares is  $64 - 49 = 15$

Step 2 – the difference between the number and the smallest perfect square is  $59 - 49 = 10$

Step 3 – divide the two difference  $\frac{10}{15} \approx 0.7$

Step 4 – add our answer from step 3 to the square root of the smallest perfect square  $7 + 0.7 = 7.7$ .

So  $\sqrt{59} \approx 7.7$

### Applications and formulas:

**Example:** Find the range of a submarine periscope that is 9ft above the surface of the water. Use the formula  $R = 1.4\sqrt{h}$ , where R is the range in miles and  $h$  is the height in feet of the periscope above the surface of the water. Round to the nearest hundredth.

We start with our equation

$$R = 1.4\sqrt{h}$$

and replace the value of 9 for  $h$

$$R = 1.4\sqrt{9}$$

Now we take the square root of 9

$$R = 1.4(3)$$

Performing the multiplication gives us

$$R = 4.2$$

So now we know that the range is 4.20 miles.