Introductory Algebra
Chapter 5 Review

Objective [5.1a] Factor monomials.		
Brief Procedure	**Example**	**Practice Exercise**
Find two monomials whose product is equivalent to the original monomial.	Find three factorizations of $24x^5$. 1) $24x^5 = (3 \cdot 8)(x \cdot x^4) = (3x)(8x^4)$ 2) $24x^5 = (2 \cdot 12)(x^2 \cdot x^3) = (2x^2)(12x^3)$ 3) $24x^5 = (-4)(-6)x^5 = (-4)(-6x^5)$ There are other factorizations as well.	1. Which is not a factorization of $32x^6$? A. $(-1)(-32x^6)$ B. $(4x^3)(8x^3)$ C. $(-16x^2)(2x^4)$ D. $(32)(x^6)$
Objective [5.1b] Factor polynomials where the terms have a common factor, factoring out the largest common factor.		
Brief Procedure	**Example**	**Practice Exercise**
Find the largest factor common to all the terms of the polynomial. Then use the distributive law to express the polynomial as a product where one factor is the largest common factor.	Factor $18x^5 - 9x^3 + 27x^2$. Although there are many factors common to the three terms, the *largest* common factor is $9x^2$. $18x^5 - 9x^3 + 27x^2$ $= (9x^2)(2x^3) - (9x^2)(x) + (9x^2)(3)$ $= 9x^2(2x^3 - x + 3)$	2. Factor $24y^8 + 16y^6 - 8y^4$, factoring out the largest common factor. A. $2y(12y^7 + 8y^5 - 4y^3)$ B. $4y^3(6y^5 + 4y^3 - 2y)$ C. $8y^4(3y^4 + 2y^2)$ D. $8y^4(3y^4 + 2y^2 - 1)$
Objective [5.1c] Factor certain expressions with four terms using factoring by grouping.		
Brief Procedure	**Example**	**Practice Exercise**
Group the terms into two pairs. Factor each group and then factor the common factor out of the resulting expression.	Factor $6x^3+9x^2-8x-12$ by grouping. $6x^3 + 9x^2 - 8x - 12$ $= (6x^3 + 9x^2) + (-8x - 12)$ $= 3x^2(2x + 3) - 4(2x + 3)$ $= (3x^2 - 4)(2x + 3)$	3. Factor x^3-3x^2+2x-6 by grouping. A. One factor is $(x + 2)$. B. One factor is $(x^2 + 2)$. C. One factor is $(x^2 - 3)$. D. One factor is $(x + 3)$.

Objective [5.2a] Factor trinomials of the type $x^2 + bx + c$ by examining the constant term c.

Brief Procedure	Example	Practice Exercise
1. Arrange the trinomial in descending order. 2. Use a trial-and-error process that looks for factors of c whose sum is b. 3. If c is positive, the signs of the factors are the same as the sign of b. 4. If c is negative, one factor is positive and the other is negative. If the sum of two factors is the opposite of b, changing the sign of each factor will give the desired factors whose sum is b. 5. Check by multiplying.	Factor $x^2 - 2x - 15$. Since the constant term, -15, is negative, we look for a factorization of -15 in which one factor is positive and one factor is negative. The sum of the factors must be the coefficient of the middle term, -2, so the negative factor must have the larger absolute value. The possible pairs of factors that meet these criteria are 1, -15 and 3, -5. The numbers we need are 3 and -5. $x^2 - 2x - 15 = (x + 3)(x - 5)$.	4. Factor $x^2 - 9x + 8$. A. One factor is $(x + 1)$. B. One factor is $(x - 1)$. C. One factor is $(x + 8)$. D. One factor is $(x - 4)$.

Objective [5.3a] Factor trinomials of the type $ax^2 + bx + c, a \neq 1$, using FOIL.

Brief Procedure	Example	Practice Exercise
1. Factor out the largest common factor. 2. Find the **F**irst terms whose product is ax^2. 3. Find two **L**ast terms whose product is c. 4. Repeat steps (2) and (3) until a combination is found for which the sum of the **O**uter and **I**nner products is bx.	Factor $2y^3 + 5y^2 - 3y$. 1. Factor out the largest common factor, y: $y(2y^2 + 5y - 3)$. Now we factor $2y^2 + 5y - 3$. 2. Because $2y^2$ factors as $2y \cdot y$, we have this possibility for a factorization: $(2y+ \quad)(y+ \quad)$. 3. There are two pairs of factors of -3 and each can be written in two ways: $3, -1 \quad -3, 1$ and $-1, 3 \quad 1, -3$. 4. From steps (2) and (3) we see that there are 4 possibilities for factorizations. We look for Outer and Inner products for which the sum is the middle term, $5y$. We try some possibilities. $(2y + 3)(y - 1) = 2y^2 + y - 3$ $(2y - 1)(y + 3) = 2y^2 + 5y - 3$ The factorization of $2y^2 + 5y - 3$ is $(2y - 1)(y + 3)$. We must include the common factor to get a factorization of the original trinomial. $2y^3 + 5y^2 - 3y = y(2y - 1)(y + 3)$	5. Factor $6z^2 + 14z + 4$. A. $(3z + 1)(z + 2)$ B. $2(3z + 1)(z + 2)$ C. $(6z + 1)(z + 6)$ D. $(3z + 2)(2z + 3)$

Objective [5.4a] Factor trinomials of the type $ax^2 + bx + c, a \neq 1$, by splitting the middle term and using grouping.

Brief Procedure	Example	Practice Exercise
1. Factor out a common factor , if any. 2. Multiply the leading coefficient a and the constant c. 3. Try to factor the product ac so that the sum of the factors is b. That is, find integers p and q such that $pq = ac$ and $p + q = b$. 4. Split the middle term. That is, write it as a sum using the factors found in step (3). 5. Then factor by grouping.	Factor $5x^2 + 7x - 6$ by grouping. 1. There is no common factor (other than 1 or -1). 2. Multiply the leading coefficient 5 and the constant, -6: $5(-6) = -30$. 3. Look for a factorization of -30 in which the sum of the factors is the coefficient of the middle term, 7. The numbers we need are 10 and -3. 4. Split the middle term, writing it as a sum or difference using the factors found in step (3). $7x = 10x - 3x$ 5. Factor by grouping. $5x^2 + 7x - 6$ $= 5x^2 + 10x - 3x - 6$ $= 5x(x + 2) - 3(x + 2)$ $= (5x - 3)(x + 2)$	6. Factor $8x^2 - 2x - 1$ by grouping. A. One factor is $(x - 1)$. B. One factor is $(2x - 1)$. C. One factor is $(4x - 1)$. D. One factor is $(8x - 1)$.

Objective [5.5a] Recognize trinomial squares.

Brief Procedure	Example	Practice Exercise
In order for an expression to be a trinomial square (that is, of the type $A^2 + 2AB + B^2$ or $A^2 - 2AB + B^2$): a) Two terms, A^2 and B^2, must be squares. b) There must be no minus sign before A^2 or B^2. c) The middle term must be $2 \cdot A \cdot B$ or $-2 \cdot A \cdot B$.	Determine whether each of the following is a trinomial square. a) $x^2 - 4x - 4$ b) $9x^2 + 1 + 6x$ a) x^2 and 4 are squares, but there is a minus sign before 4. This is not a trinomial square. b) Write the trinomial in descending order: $9x^2 + 6x + 1$. $9x^2$ and 1 are squares. There is no minus sign before $9x^2$ or 1. The middle term, $6x$, is $2 \cdot 3x \cdot 1$. Thus $9x^2 + 1 + 6x$ is a trinomial square.	7. Determine whether $y^2 + 32y + 16$ is a trinomial square. A.Yes B. No

Objective [5.5b] Factor trinomial squares.		
Brief Procedure	Example	Practice Exercise
Use the following equations: $A^2 + 2AB + B^2 = (A + B)^2$, $A^2 - 2AB + B^2 = (A - B)^2$	Factor $4x^2 - 12x + 9$. $4x^2 - 12x + 9$ $= (2x)^2 - 2 \cdot 2x \cdot 3 + 3^2$ $= (2x - 3)^2$	8. Factor $16x^2 + 8x + 1$. A. $(2x + 1)^2$ B. $(4x + 1)^2$ C. $(8x + 1)^2$ D. $(8x - 1)^2$

Objective [5.5c] Recognize differences of squares.		
Brief Procedure	Example	Practice Exercise
a) There must be two expressions, both squares. b) The expressions must have different signs.	Determine whether each of the following is a difference of squares. a) $16t^4 - 9$ b) $25x^2 - 3$ a) The expressions $16t^4$ and 9 are squares: $16t^4 = (4t^2)^2$ and $9 = 3^2$. The expressions have different signs. Thus, $16t^4 - 9$ is a difference of squares. b) 3 is not a square, so $25x^2 - 3$ is not a difference of squares.	9. Determine whether $1 - 36y^6$ is a difference of squares. A.Yes B. No

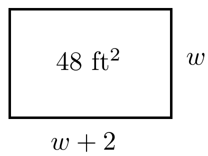
Objective [5.5d] Factor differences of squares, being careful to factor completely.		
Brief Procedure	Example	Practice Exercise
Use the equation $A^2 - B^2 = (A + B)(A - B)$.	Factor $t^5 - t$. $t^5 - t = t(t^4 - 1)$ $= t(t^2 + 1)(t^2 - 1)$ $= t(t^2 + 1)(t + 1)(t - 1)$	10. Factor $12 - 27x^2$ completely. A.$3(4 - 9x^2)$ B. $(2 + 3x)(2 - 3x)$ C. $3(2 + 3x)(2 - 3x)$ D. $3(3x + 2)(3x - 2)$

Objective [5.6a] Factor polynomials completely using any of the methods considered in this chapter.		
Brief Procedure	Example	Practice Exercise
a) Always look for a common factor. If there is one, factor out the largest common factor. b) Then look at the number of terms. *Two terms:* Determine whether you have a difference of squares. Do not try to factor a sum of squares: $A^2 + B^2$. *Three terms:* Determine whether the trinomial is a square. If it is, factor accordingly. If not, try trial and error, using FOIL or grouping. *Four terms:* Try factoring by grouping. c) *Always factor completely.* If a factor with more than one term can still be factored, you should factor it. When no factor can be factored further, you have finished.	Factor $2y^3 - 12y^2 + 18y$ completely. a) We look for a common factor. $2y^3 - 12y^2 + 18y = 2y(y^2 - 6y + 9)$ b) The factor $y^2 - 6y + 9$ has three terms and is a trinomial square. We factor it. $2y(y^2 - 6y + 9)$ $= 2y(y^2 - 2 \cdot y \cdot 3 + 3^2)$ $= 2y(y - 3)^2$	11. Factor $15x^2 + 5x - 20$ completely. A. One factor is $(3x + 4)$. B. One factor is $(3x - 4)$. C. One factor is $(5x - 5)$. D. One factor is $(15x + 20)$.

Objective [5.7a] Solve equations (already factored) using the principle of zero products.		
Brief Procedure	Example	Practice Exercise
An equation $ab = 0$ is true if and only if $a = 0$ is true or $b = 0$ is true, or both are true. (A product is 0 if and only if one or both of the factors is 0.)	Solve: $(4x - 3)(x + 2) = 0$. $(4x - 3)(x + 2) = 0$ $4x - 3 = 0$ *or* $x + 2 = 0$ $4x = 3$ *or* $x = -2$ $x = \frac{3}{4}$ *or* $x = -2$ The solutions are $\frac{3}{4}$ and -2.	12. Solve: $x(6x + 5) = 0$. A. 0 B. $-\frac{5}{6}$ C. $0, -\frac{5}{6}$ D. $0, -\frac{6}{5}$

Objective [5.7b] Solve quadratic equations by factoring and then using the principle of zero products.		
Brief Procedure	Example	Practice Exercise
Write the equation in the form $ax^2 + bx + c = 0$, factor $ax^2 + bx + c$, and then use the principle of zero products.	Solve: $x^2 + 2x = 24$. $x^2 + 2x = 24$ $x^2 + 2x - 24 = 0$ $(x + 6)(x - 4) = 0$ $x + 6 = 0$ *or* $x - 4 = 0$ $x = -6$ *or* $x = 4$ The solutions are -6 and 4.	13. Solve: $16x^2 = 49$. A. $\frac{7}{4}, -\frac{7}{4}$ B. $\frac{4}{7}, -\frac{4}{7}$ C. $7, -7$ D. $4, -4$

Objective [5.8a] Solve applied problems involving quadratic equations that can be solved by factoring.

Brief Procedure	Example	Practice Exercise
Use the five-step problem solving process.	The length of a rectangular rug is 2 ft greater than the width. The area of the rug is 48 ft^2. Find the length and width. 1. *Familiarize.* We make a drawing. Let w = the width of the rug. Then the length is $w+2$.  Recall that the area of a rectangle is length × width. 2. *Translate.* We reword the problem. Length × width is 48 ft^2. $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$ $(w+2) \times w = 48$ 3. *Solve.* We solve the equation. $(w+2) \times w = 48$ $w^2 + 2w = 48$ $w^2 + 2w - 48 = 0$ $(w+8)(w-6) = 0$ $w+8=0$ *or* $w-6=0$ $w=-8$ *or* $w=6$ 4. *Check.* The width of a rectangle cannot be negative, so -8 cannot be a solution. Suppose the width is 6 ft. Then the length is $6+2$, or 8 ft and the area is $6 \cdot 8$, or 48 ft^2. These numbers check in the original problem. 5. *State.* The length is 8 ft and the width is 6 ft.	14. The height of a triangle is 4 cm greater than the base. The area is 30 cm^2. Find the height and the base. A. The height is 6 cm. B. The height is 10 cm. C. The base is 10 cm. D. The base is 14 cm.